

## Scaling analysis of the viscoelastic response of linear polymers

Mohamed F., Flämig M., Hofmann M., Heymann L., Willner L., Fatkullin N., Aksel N., Rössler E.  
*Kazan Federal University, 420008, Kremlevskaya 18, Kazan, Russia*

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### Abstract

© 2018 Author(s). Viscoelastic response in terms of the complex shear modulus  $G^*(\omega)$  of the linear polymers poly(ethylene-alt-propylene), poly(isoprene), and poly(butadiene) is studied for molar masses ( $M$ ) from 3k up to 1000k and over a wide temperature range starting from the glass transition temperature  $T_g$  (174 K–373 K). Master curves  $G'(\omega\tau_\alpha)$  and  $G''(\omega\tau_\alpha)$  are constructed for the polymer-specific relaxation. Segmental relaxation occurring close to  $T_g$  is independently addressed by single spectra. Altogether, viscoelastic response is effectively studied over 14 decades in frequency. The structural relaxation time  $\tau_\alpha$  used for scaling is taken from dielectric spectra. We suggest a derivative method for identifying the different power-law regimes and their exponents along  $G''(\omega\tau_\alpha) \propto \omega^\epsilon$ . The exponent  $\epsilon'' = \epsilon''(\omega\tau_\alpha) = d \ln G''(\omega\tau_\alpha) / d \ln(\omega\tau_\alpha)$  reveals more details compared to conventional analyses and displays high similarity among the polymers. Within a simple scaling model, the original tube-reptation model is extended to include contour length fluctuations (CLFs). The model reproduces all signatures of the quantitative theory by Likhtman and McLeish. The characteristic times and power-law exponents are rediscovered in  $\epsilon''(\omega\tau_\alpha)$ . The high-frequency flank of the terminal relaxation closely follows the prediction for CLF ( $\epsilon'' = -0.25$ ), i.e.,  $G''(\omega) \propto \omega^{-0.21 \pm 0.02}$ . At lower frequencies, a second regime with lower exponent  $\epsilon''$  is observed signaling the crossover to coherent reptation. Application of the full Likhtman-McLeish calculation provides a quantitative interpolation of  $\epsilon''(\omega\tau_\alpha)$  at frequencies below those of the Rouse regime. The derivative method also allows identifying the entanglement time  $\tau_e$ . However, as the exponent in the Rouse regime ( $\omega\tau_e > 1$ ) varies along  $\epsilon_{\text{Rouse}} = 0.66 \pm 0.04$  (off the Rouse prediction  $\epsilon_{\text{Rouse}} = 0.5$ ) and that at  $\omega\tau_e < 1$  is similar, only a weak manifestation of the crossover at  $\tau_e$  is found at highest  $M$ . Yet, calculating  $\tau_e/\tau_\alpha = (M/M_0)^2$ , we find good agreement among the polymers when discussing  $\epsilon''(\omega\tau_e)$ . The terminal relaxation time  $\tau_t$  is directly read off from  $\epsilon''(\omega\tau_\alpha)$ . Plotting  $\tau_t/\tau_\alpha$  as a function of  $Z = M/M_e$ , we find universal behavior as predicted by the TR model. The  $M$  dependence crosses over from an exponent significantly larger than 3.0 at intermediate  $M$  to an exponent approaching 3.0 at highest  $M$  in agreement with previous reports. The frequency of the minimum in  $G''(\omega\tau_\alpha)$  scales as  $\tau_{\min} \propto M^{1.0 \pm 0.1}$ . An  $M$ -independent frequency marks the crossover to glassy relaxation at the highest frequencies. Independent of the amplitude of  $G''(\omega)$ , which may be related to sample-to-sample differences, the derivative method is a versatile tool to provide a detailed phenomenological analysis of the viscoelastic response of complex liquids.

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